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$f(R)$ -Gravity: “Einstein Frame” Lagrangian Formulation, Non-Standard Black Holes and QCD-like Confinement/Deconfinement

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Abstract We consider $f(R) = R + R^2$ gravity interacting with a dilaton and a special non-standard form of nonlinear electrodynamics containing a square-root of ordinary Maxwell Lagrangian. In flat spacetime the latter arises due to a spontaneous breakdown of scale symmetry and produces an effective charge-confining potential. In the $R + R^2$ gravity case, upon deriving the explicit form of the equivalent *local* “Einstein frame” Lagrangian action, we find several physically relevant features due to the combined effect of the gauge field and gravity nonlinearities such as: appearance of dynamical effective gauge couplings and *confinement-deconfinement transition effect* as functions of the dilaton vacuum expectation value; new mechanism for dynamical generation of cosmological constant; deriving non-standard black hole solutions carrying additional constant vacuum radial electric field and with non-asymptotically flat “hedge-hog”-type spacetime asymptotics.

1 Introduction

$f(R)$ -gravity models (where $f(R)$ is a nonlinear function of the scalar curvature R and, possibly, of other higher-order invariants of the Riemann curvature tensor $R_{\lambda\mu\nu}^{\kappa}$) are attracting a lot of interest as possible candidates to cure problems in the standard cosmological models related to dark matter and dark energy. For a recent review of $f(R)$ -gravity see e.g. [1] and references therein ¹.

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¹ The first R^2 -model (within the second order formalism), which was proposed as the first inflationary model, appeared in Ref.[2].

In the present contribution we consider $f(R)$ -gravity coupled to scalar dilaton ϕ and most notably – to a *non-standard nonlinear gauge field system containing $\sqrt{-F^2}$* (square-root of standard Maxwell kinetic term; see Refs.[3, 4, 5]), which is known to produce confining effective potential among quantized charged fermions in flat spacetime [4].

We describe in some detail the explicit derivation of the effective Lagrangian governing the $f(R)$ -gravity dynamics in the so called ‘‘Einstein frame’’. The latter means that in terms of an appropriate *conformal rescaling* of the original spacetime metric $g_{\mu\nu} \rightarrow h_{\mu\nu} = f'_R g_{\mu\nu}$ (where $f'_R = df/dR$) the pertinent gravity part of the effective action assumes the standard form of Einstein-Hilbert action ($\sim R(h)$).

Our main goal is to derive a *local* ‘‘Einstein frame’’ effective Lagrangian *for the matter fields* as well – this is explicitly done for ‘‘ $R + R^2$ -gravity’’.

Namely, in the special case of $f(R) = R + \alpha R^2$ the passage to the ‘‘Einstein frame’’ entails non-trivial modifications in the effective matter Lagrangian, which *in combination with the special ‘‘square-root’’ gauge field nonlinearity* triggers various physically interesting effects:

- (i) appearance of dynamical effective gauge couplings and *confinement-deconfinement transition effect* as functions of the dilaton vacuum expectation value (v.e.v.);
- (ii) new mechanism for dynamical generation of cosmological constant;
- (iii) non-standard black hole solutions carrying a constant vacuum radial electric field (such electric fields do not exist in ordinary Maxwell electrodynamics) and exhibiting non-asymptotically flat ‘‘hedgehog’’-type [6] spacetime asymptotics;
- (iv) the above non-standard black holes are shown to obey the first law of black hole thermodynamics;
- (v) obtaining new ‘‘tubelike universe’’ solutions of Levi-Civita-Bertotti-Robinson type $\mathcal{M}_2 \times S^2$ [7].

In addition, as shown in Ref.[8] coupling of the gravity/nonlinear gauge field system to *lightlike* branes produces ‘‘charge-’’hiding’’ and charge-confining ‘‘thin-shell’’ wormhole solutions displaying QCD-like confinement.

The main motivation for including the nonlinear gauge field term $\sqrt{-F^2}$ comes from the works [9] of G. ‘t Hooft, who has shown that in any effective quantum gauge theory, which is able to describe linear confinement phenomena, the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region (the latter appearing as an ‘‘infrared counterterm’’).

The simplest way to realize ‘t Hooft’s ideas in flat spacetime has been worked out in Refs.[3, 4, 5] where the following nonlinear modification of Maxwell action has been proposed:

$$S = \int d^4x L(F^2) \quad , \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \quad , \quad (1)$$

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad .$$

The square root of the Maxwell kinetic term naturally arises as a result of *spontaneous breakdown of scale symmetry* of the original scale-invariant Maxwell action

with f_0 appearing as an integration constant responsible for the latter spontaneous breakdown.

For static field configurations the model (1) yields an electric displacement field $\mathbf{D} = \mathbf{E} - \frac{f_0}{\sqrt{2}} \frac{\mathbf{E}}{|\mathbf{E}|}$ and the corresponding energy density turns out to be $\frac{1}{2}\mathbf{E}^2 = \frac{1}{2}|\mathbf{D}|^2 + \frac{f_0}{\sqrt{2}}|\mathbf{D}| + \frac{1}{4}f_0^2$, so that it indeed contains a term linear w.r.t. $|\mathbf{D}|$ as predicted by the phenomenological theory of 't Hooft.

The *non-standard nonlinear* gauge field system (1) produces in flat spacetime [4], when coupled to quantized fermions, a confining effective potential $V(r) = -\frac{\beta}{r} + \gamma r$ (Coulomb plus linear one with $\gamma \sim f_0$) which is of the form of the well-known ‘‘Cornell’’ potential [10] in the phenomenological description of quarkonium systems in QCD.

2 $f(R)$ -Gravity in the ‘‘Einstein Frame’’

Consider $f(R) = R + \alpha R^2 + \dots$ gravity (possibly with a bare cosmological constant Λ_0) coupled to a dilaton ϕ and a nonlinear gauge field system containing $\sqrt{-F^2}$:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi} \left(f(R(g, \Gamma)) - 2\Lambda_0 \right) + L(F^2(g)) + L_D(\phi, g) \right], \quad (2)$$

$$L(F^2(g)) = -\frac{1}{4e^2} F^2(g) - \frac{f_0}{2} \sqrt{-F^2(g)}, \quad (3)$$

$$F^2(g) \equiv F_{\kappa\lambda} F_{\mu\nu} g^{\kappa\mu} g^{\lambda\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (4)$$

$$L_D(\phi, g) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi). \quad (5)$$

where $R(g, \Gamma) = R_{\mu\nu}(\Gamma) g^{\mu\nu}$ and $R_{\mu\nu}(\Gamma)$ is the Ricci curvature in the first order (Palatini) formalism, *i.e.*, the spacetime metric $g_{\mu\nu}$ and the affine connection $\Gamma_{\nu\lambda}^\mu$ are *a priori* independent variables.

The equations of motion resulting from the action (2) read:

$$R_{\mu\nu}(\Gamma) = \frac{1}{f'_R} \left[8\pi T_{\mu\nu} + \frac{1}{2} f(R(g, \Gamma)) g_{\mu\nu} \right], \quad (6)$$

$$f'_R \equiv \frac{df(R)}{dR}, \quad \nabla_\lambda (\sqrt{-g} f'_R g^{\mu\nu}) = 0, \quad (7)$$

$$\partial_\nu \left(\sqrt{-g} \left[1/e^2 - \frac{f_0}{\sqrt{-F^2(g)}} \right] F_{\kappa\lambda} g^{\mu\kappa} g^{\nu\lambda} \right) = 0. \quad (8)$$

The total energy-momentum tensor is given by:

$$T_{\mu\nu} = \left[L(F^2(g)) + L_D(\phi, g) - \frac{1}{8\pi} \Lambda_0 \right] g_{\mu\nu} + \left(1/e^2 - \frac{f_0}{\sqrt{-F^2(g)}} \right) F_{\mu\kappa} F_{\nu\lambda} g^{\kappa\lambda} + \partial_\mu \phi \partial_\nu \phi. \quad (9)$$

Eq.(7) leads to the relation $\nabla_\lambda (f'_R g_{\mu\nu}) = 0$ and thus it implies transition to the physical ‘‘Einstein frame’’ metrics $h_{\mu\nu}$ via conformal rescaling of the original metric $g_{\mu\nu}$ [11]:

$$g_{\mu\nu} = \frac{1}{f'_R} h_{\mu\nu} \quad , \quad \Gamma_{\nu\lambda}^\mu = \frac{1}{2} h^{\mu\kappa} (\partial_\nu h_{\lambda\kappa} + \partial_\lambda h_{\nu\kappa} - \partial_\kappa h_{\nu\lambda}) . \quad (10)$$

Using (10) the $f(R)$ -gravity equations of motion (6) can be rewritten in the form of *standard* Einstein equations:

$$R_{\mu\nu} = 8\pi \left(T_{\text{eff}\mu\nu} - \frac{1}{2} g_{\mu\nu} T_{\text{eff}} \right) \quad (11)$$

where $T_{\text{eff}} = g^{\mu\nu} T_{\text{eff}\mu\nu}$ and with effective energy-momentum tensor $T_{\text{eff}\mu\nu}$ of the following form:

$$T_{\text{eff}\mu\nu} = \frac{1}{f'_R} \left[T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T \right] - \frac{1}{32\pi} g_{\mu\nu} R(T) . \quad (12)$$

Here $T \equiv g^{\mu\nu} T_{\mu\nu}$, $R(T)$ is the original scalar curvature determined as function of T from the trace of Eq.(6):

$$8\pi T = R f'_R - 2f(R) , \quad (13)$$

and everywhere in (11)–(13) $g_{\mu\nu}$ and $\Gamma_{\nu\lambda}^\mu$ are understood as functions of the ‘‘Einstein frame’’ metric $h_{\mu\nu}$ (10).

3 Einstein-Frame Effective Action

We are now looking for an effective action $S_{\text{eff}} = \int d^4x \sqrt{-h} \left[\frac{1}{16\pi} R(h) + L_{\text{eff}} \right]$, where $R(h)$ is the standard Ricci scalar of the ‘‘Einstein frame’’ metric $h_{\mu\nu}$ and $L_{\text{eff}} \equiv L_{\text{eff}}(h_{\mu\nu}, A_\mu, \phi)$ is a *local function* of the corresponding (matter) fields and of their derivatives, such that it produces in the standard way the original $f(R)$ -gravity equations of motion (6) (or equivalently (11)–(13)). L_{eff} will also include an *effective* cosmological constant term irrespective of the presence or absence of a bare cosmological constant Λ_0 in the original $f(R)$ -gravity action (2).

L_{eff} must obey the following relation to the ‘‘Einstein frame’’ effective energy-momentum tensor (12):

$$T_{\text{eff}\mu\nu} = h_{\mu\nu} L_{\text{eff}} - 2 \frac{\partial L_{\text{eff}}}{\partial h^{\mu\nu}} . \quad (14)$$

Henceforth we will explicitly consider the simplest nonlinear $f(R)$ -gravity case: $f(R) = R + \alpha R^2$ (so that $f'_R = 1 + 2\alpha R$).

The generic form of the matter Lagrangian reads:

$$L_m = L^{(0)} + L^{(1)}(g) + L^{(2)}(g), \quad (15)$$

where the superscripts indicate homogeneity degree w.r.t. $g^{\mu\nu}$. Solving relation (14) by taking into account the conformal rescaling of $g_{\mu\nu}$ (10) and the homogeneity relation (15) we find the following *local* effective ‘‘Einstein frame’’ matter Lagrangian:

$$L_{\text{eff}} = \frac{1}{1 - 64\pi\alpha L^{(0)}} \left[L^{(0)} + L^{(1)}(h)(1 + 16\pi\alpha L^{(1)}(h)) \right] + L^{(2)}(h), \quad (16)$$

where now the superscripts indicate homogeneity degree w.r.t. $h^{\mu\nu}$.

Explicitly, in the case of $R + R^2$ -gravity/nonlinear-gauge-field/dilaton system (2)–(5) we have (using shortcut notations $F^2(h) \equiv F_{\kappa\lambda}F_{\mu\nu}h^{\kappa\mu}h^{\lambda\nu}$ and $X(\phi, h) \equiv -\frac{1}{2}h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$):

$$L_{\text{eff}} = -\frac{1}{4e_{\text{eff}}^2(\phi)}F^2(h) - \frac{1}{2}f_{\text{eff}}(\phi)\sqrt{-F^2(h)} + \frac{X(\phi, h)(1 + 16\pi\alpha X(\phi, h)) - V(\phi) - \Lambda_0/8\pi}{1 + 8\alpha(8\pi V(\phi) + \Lambda_0)} \quad (17)$$

with the dynamically generated dilaton ϕ -dependent couplings:

$$\frac{1}{e_{\text{eff}}^2(\phi)} = \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha(8\pi V(\phi) + \Lambda_0)}, \quad (18)$$

$$f_{\text{eff}}(\phi) = f_0 \frac{1 + 32\pi\alpha X(\phi, h)}{1 + 8\alpha(8\pi V(\phi) + \Lambda_0)}. \quad (19)$$

Here is an important observation about the effective action:

$$S_{\text{eff}} = \int d^4x \sqrt{-h} \left[\frac{R(h)}{16\pi} + L_{\text{eff}}(h, A, \phi) \right]. \quad (20)$$

Even if ordinary kinetic Maxwell term $-\frac{1}{4}F^2$ is absent in the original system ($e^2 \rightarrow \infty$ in (3)), such term is nevertheless *dynamically generated* in the ‘‘Einstein frame’’ action (17)–(20) – an explicit manifestation of the *combined effect* of gravitational and gauge field nonlinearities (αR^2 and $-\frac{f_0}{2}\sqrt{-F^2}$):

$$S_{\text{maxwell}} = -4\pi\alpha f_0^2 \int d^4x \sqrt{-h} \frac{F_{\kappa\lambda}F_{\mu\nu}h^{\kappa\mu}h^{\lambda\nu}}{1 + 8\alpha(8\pi V(\phi) + \Lambda_0)}. \quad (21)$$

4 Confinement/Deconfinement Phases

In what follows we consider constant dilaton ϕ extremizing the effective Lagrangian (17) (*i.e.*, the dilaton kinetic term $X(\phi, h)$ will be ignored in the sequel):

$$L_{\text{eff}} = -\frac{1}{4e_{\text{eff}}^2(\phi)}F^2(h) - \frac{1}{2}f_{\text{eff}}(\phi)\sqrt{-F^2(h)} - V_{\text{eff}}(\phi), \quad (22)$$

$$V_{\text{eff}}(\phi) = \frac{V(\phi) + \frac{\Lambda_0}{8\pi}}{1 + 8\alpha(8\pi V(\phi) + \Lambda_0)}, \quad (23)$$

$$f_{\text{eff}}(\phi) = \frac{f_0}{1 + 8\alpha(8\pi V(\phi) + \Lambda_0)}, \quad (24)$$

$$\frac{1}{e_{\text{eff}}^2(\phi)} = \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha(8\pi V(\phi) + \Lambda_0)}. \quad (25)$$

Here we uncover the following important property: *the dynamical couplings and the effective potential are extremized simultaneously*, which is an explicit realization of the so called “least coupling principle” of Damour-Polyakov [12]:

$$\frac{\partial f_{\text{eff}}}{\partial \phi} = -64\pi\alpha f_0 \frac{\partial V_{\text{eff}}}{\partial \phi}, \quad \frac{\partial}{\partial \phi} \frac{1}{e_{\text{eff}}^2} = -(32\pi\alpha f_0)^2 \frac{\partial V_{\text{eff}}}{\partial \phi} \rightarrow \frac{\partial L_{\text{eff}}}{\partial \phi} \sim \frac{\partial V_{\text{eff}}}{\partial \phi}. \quad (26)$$

Therefore, at the extremum of L_{eff} (22) ϕ must satisfy:

$$\frac{\partial V_{\text{eff}}}{\partial \phi} = \frac{V'(\phi)}{[1 + 8\alpha(\kappa^2 V(\phi) + \Lambda_0)]^2} = 0. \quad (27)$$

There are two generic cases:

(A) *Confining phase*: Eq.(27) is satisfied for some finite value ϕ_0 extremizing the original potential $V(\phi)$: $V'(\phi_0) = 0$.

(B) *Deconfinement phase*: For polynomial or exponentially growing original potential $V(\phi)$, so that $V(\phi) \rightarrow \infty$ when $\phi \rightarrow \infty$, we have:

$$\frac{\partial V_{\text{eff}}}{\partial \phi} \rightarrow 0, \quad V_{\text{eff}}(\phi) \rightarrow \frac{1}{64\pi\alpha} = \text{const} \quad \text{when } \phi \rightarrow \infty, \quad (28)$$

i.e., for sufficiently large values of ϕ we find a “flat region” in the effective potential V_{eff} . This “flat region” triggers a *transition from confining to deconfinement dynamics*.

Namely, in the confining phase (A) (generic minimum ϕ_0 of the effective dilaton potential) we have shown in [13] that the following *confining potential* (linear w.r.t. r) acts on charged test point-particles:

$$\frac{\sqrt{2}\mathcal{E}|q_0|}{m_0^2} e_{\text{eff}}(\phi_0) f_{\text{eff}}(\phi_0) r, \quad (29)$$

where \mathcal{E}, m_0, q_0 are energy, mass and charge of the test particle.

In the deconfinement phase (B) (“flat-region” of the effective dilaton potential) we have:

$$f_{\text{eff}} \rightarrow 0, \quad e_{\text{eff}}^2 \rightarrow e^2 \quad (30)$$

and the effective gauge field Lagrangian (22) reduces to the ordinary *non-confining* one (the “square-root” term $\sqrt{-F^2}$ vanishes):

$$L_{\text{eff}}^{(0)} = -\frac{1}{4e^2}F^2(h) - \frac{1}{64\pi\alpha} \quad (31)$$

with an *induced* cosmological constant $\Lambda_{\text{eff}} = 1/8\alpha$, which is *completely independent* of the bare cosmological constant Λ_0 .

5 Non-Standard Black Holes and New “Tubelike” Solutions

From the effective Einstein-frame action (20) with L_{eff} as in (22) we find *non-standard* Reissner-Nordström-(anti-)de-Sitter-type black hole solutions in the confining phase (ϕ_0 – generic minimum of the effective dilaton potential (23); $e_{\text{eff}}(\phi)$), $f_{\text{eff}}(\phi)$ as in (24)–(25)):

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (32)$$

$$A(r) = 1 - \sqrt{8\pi}|Q|f_{\text{eff}}(\phi_0)e_{\text{eff}}(\phi_0) - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\text{eff}}(\phi_0)}{3}r^2, \quad (33)$$

with *dynamically generated* cosmological constant:

$$\Lambda_{\text{eff}}(\phi_0) = \frac{\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2}{1 + 8\alpha(\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2)}. \quad (34)$$

The black hole’s static spherically symmetric electric field contains apart from the Coulomb term an *additional constant radial “vacuum” piece* responsible for confinement (let us stress again that constant vacuum radial electric fields do not exist in ordinary Maxwell electrodynamics):

$$|F_{0r}| = |\mathbf{E}_{\text{vac}}| + \frac{|Q|}{\sqrt{4\pi}r^2} \left(\frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha(8\pi V(\phi_0) + \Lambda_0)} \right)^{-\frac{1}{2}} \quad (35)$$

$$|\mathbf{E}_{\text{vac}}| \equiv \left(\frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha(8\pi V(\phi_0) + \Lambda_0)} \right)^{-1} \frac{f_0/\sqrt{2}}{1 + 8\alpha(8\pi V(\phi_0) + \Lambda_0)}. \quad (36)$$

For the special value of ϕ_0 where $\Lambda_{\text{eff}}(\phi_0) = 0$ we obtain Reissner-Nordström-type black hole with a “hedgehog” [6] *non-flat-spacetime* asymptotics:

$$A(r) \rightarrow 1 - \sqrt{8\pi}|Q|f_{\text{eff}}(\phi_0)e_{\text{eff}}(\phi_0) \neq 1 \text{ for } r \rightarrow \infty.$$

Further we obtain Levi-Civita-Bertotti-Robinson (LCBR) [7] type “tubelike” spacetime solutions with geometries $\mathcal{M}_2 \times S^2$ (\mathcal{M}_2 – 2-dimensional manifold) with metric of the form:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2(d\theta^2 + \sin^2\theta d\phi^2), \quad -\infty < \eta < \infty, \quad (37)$$

and constant vacuum “radial” electric field $|F_{0\eta}| = |\mathbf{E}_{\text{vac}}|$, where the size of the S^2 -factor is given by (using short-hand $\Lambda(\phi_0) \equiv 8\pi V(\phi_0) + \Lambda_0$):

$$\frac{1}{r_0^2} = \frac{4\pi}{1 + 8\alpha\Lambda(\phi_0)} \left[\left(1 + 8\alpha(\Lambda(\phi_0) + 2\pi f_0^2) \right) \mathbf{E}_{\text{vac}}^2 + \frac{1}{4\pi} \Lambda(\phi_0) \right]. \quad (38)$$

There are three distinct solutions for LBCR (37) where $\mathcal{M}_2 = AdS_2, Rind_2, dS_2$ (2-dimensional anti-de Sitter, Rindler and de Sitter spaces, respectively):

(i) LBCR type solution $AdS_2 \times S^2$ for strong $|\mathbf{E}_{\text{vac}}|$:

$$A(\eta) = 4\pi K(\mathbf{E}_{\text{vac}}) \eta^2, \quad K(\mathbf{E}_{\text{vac}}) > 0, \quad (39)$$

in the metric (37), η being the Poincare patch space-like coordinate of AdS_2 , and

$$K(\mathbf{E}_{\text{vac}}) \equiv \left(1 + 8\alpha(\Lambda(\phi_0) + 2\pi f_0^2) \right) \mathbf{E}_{\text{vac}}^2 - \sqrt{2} f_0 |\mathbf{E}_{\text{vac}}| - \frac{1}{4\pi} \Lambda(\phi_0). \quad (40)$$

(ii) LBCR type solution $Rind_2 \times S^2$ when $K(\mathbf{E}_{\text{vac}}) = 0$:

$$A(\eta) = \eta \text{ for } 0 < \eta < \infty \quad \text{or} \quad A(\eta) = -\eta \text{ for } -\infty < \eta < 0 \quad (41)$$

(iii) LBCR type solution $dS_2 \times S^2$ for weak $|\mathbf{E}_{\text{vac}}|$:

$$A(\eta) = 1 - 4\pi |K(\mathbf{E}_{\text{vac}})| \eta^2, \quad K(\mathbf{E}_{\text{vac}}) < 0. \quad (42)$$

6 Thermodynamics of Non-Standard Black Holes

Consider static spherically symmetric metric $ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ with Schwarzschild-type horizon r_0 , i.e., $A(r_0) = 0$, $\partial_r A|_{r_0} > 0$ and with $A(r)$ of the general “non-standard” form:

$$A(r) = 1 - c(Q_i) - 2m/r + A_1(r; Q_i), \quad (43)$$

where Q_i are the rest of the black hole parameters apart from the mass m , and $c(Q_i)$ is generically a non-zero constant (as in (33)) responsible for a “hedgehog” [6] *non-flat spacetime asymptotics*.

The so called *surface gravity* κ proportional to Hawking temperature T_h is given by $\kappa = 2\pi T_h = \frac{1}{2} \partial_r A|_{r_0}$ (cf., e.g., [14]).

One can straightforwardly derive the first law of black hole thermodynamics for the non-standard black hole solutions with (43):

$$\delta m = \frac{1}{8\pi} \kappa \delta A_H + \tilde{\Phi}_i \delta Q_i, \quad A_H = 4\pi r_0^2, \quad \tilde{\Phi}_i = \frac{r_0}{2} \frac{\partial}{\partial Q_i} \left(A_1(r_0; Q_i) - c(Q_i) \right). \quad (44)$$

In the special case of non-standard Reissner-Nordström-(anti-)de-Sitter type black holes (32)–(34) with parameters (m, Q) the conjugate potential in (44):

$$\tilde{\Phi} = \frac{Q}{r_0} - \sqrt{2\pi} f_{\text{eff}}(\phi_0) e_{\text{eff}}(\phi_0) r_0 \equiv \frac{\sqrt{4\pi}}{e_{\text{eff}}^2(\phi_0)} A_0 \Big|_{r=r_0} \quad (45)$$

(with $e_{\text{eff}}(\phi_0)$ and $f_{\text{eff}}(\phi_0)$ as in (18)–(19)) is (up to a dilaton v.e.v.-dependent factor) the electric field potential A_0 ($F_{0r} = -\partial_r A_0$) of the nonlinear gauge system on the horizon.

7 Conclusions

In the present contribution we have uncovered a non-trivial interplay between a special gauge field non-linearity and $f(R)$ -gravity. On one hand, the inclusion of the non-standard nonlinear “square-root” Maxwell term $\sqrt{-F^2}$ is the explicit realization of the old “classic” idea of ‘t Hooft [9] about the nature of low-energy confinement dynamics. On the other hand, coupling of the nonlinear gauge theory containing $\sqrt{-F^2}$ to $f(R) = R + \alpha R^2$ gravity plus scalar dilaton leads to a variety of remarkable effects:

- Dynamical effective gauge couplings and dynamical induced cosmological constant – functions of dilaton v.e.v..
- New non-standard black hole solutions of Reissner-Nordström-(*anti*-)de-Sitter type carrying an additional constant vacuum radial electric field, in particular, non-standard Reissner-Nordström type black holes with asymptotically non-flat “hedgehog” behaviour.
- “Cornell”-type *confining* effective potential in charged test particle dynamics.
- Cumulative simultaneous effect of $\sqrt{-F^2}$ and R^2 -terms – triggering *transition from confining to deconfinement phase*. Standard Maxwell kinetic term for the gauge field $-F^2$ is *dynamically generated* even when absent in the original “bare” theory.

Furthermore, as we have shown in Ref.[8]:

- Coupling to a charged lightlike brane produces a charge-“hiding” wormhole, where a genuinely charged matter source is detected as electrically neutral by an external observer.
- Coupling to two oppositely charged lightlike brane sources produces a two-“throat” wormhole displaying a genuine QCD-like charge confinement.

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